Topology of Stimulus Space via Directed Network Persistent Homology

Samir Chowdhury, Bowen Dai, and Facundo Mémoli

Abstract

Computational methods such as transfer entropy and Granger causality have been widely applied in studying causal interactions between brain regions. The outputs of such methods are asymmetric causality matrices which can then be further processed as desired. A natural consideration in applying such a method is how it can be further analyzed to reveal the structure and organization of the input data. One tool that has recently gained popularity for its ability to detect the organization of data is persistent homology. However, standard persistent homology requires input data to be symmetric, and cannot be applied directly to asymmetric causal interaction data. We explore a recently-developed tool called Dowker Persistent Homology (DPH) that accepts any asymmetric, non-metric data as input, and test it via the following question: Given the output of a causality analysis method applied to spike trains of rodent hippocampal place cells, can one recover the topology (i.e. connectivity of locations) of the physical stimulus space? Prior results in the literature have already established a "learning time model" that uses persistent homology to recover topological information about the stimulus space of rodent place cells from their spike trains. Thus the validity of DPH can be verified if it can recover similar topological information after first passing the data through a preprocessing step that estimates causal interactions. We simulate spike train data using parameters established for the learning time model, and preprocess this data into Markov chains that capture causal interactions between place cells. We then apply DPH to these Markov chains, and are able to recover topological information about the physical stimulus space. Our results suggest that DPH can be effectively combined with causality inference methods for computational modeling.

Introduction. It is commonly accepted that an animal's awareness of its surroundings—the physical *stimulus space*—is encoded in the firing activity of *place cells* in its hippocampus [1]. Place cells are characterized by having firing patterns that are restricted to spatially localized regions called *place fields* [2]. Experimental results [3] suggest that the firing patterns or *spike trains* of place cells contribute spatial information that the brain uses to infer properties of the stimulus space. This has led to some interest in the following question: Without assuming place field information, what information can be extracted from only the spike trains of place cells? In [4], the authors used a mathematical shape analysis tool called *homology* to count the number of obstacles in an arena being explored by a rat. This idea was further developed in [5, 6], where the authors used a time-series sensitive notion of homology called *persistent homology* (PH) [7] to identify bounds on the choices of parameters (e.g. number of place cells, sizes of place fields, firing rate) with respect to which homology could correctly identify topological features (i.e. connectivity/adjacency of locations) of the stimulus space from spike train data. Specifically, the authors simulated spiking activity of rodent place cells as the animal explored arenas having one or two obstacles (the topological features). The ground truth to be recovered was the number of obstacles in each arena.

A different mode of information that one may wish to recover from spike train data is the structure and interaction of the neurons themselves. Specifically, one may wish to recover causal interactions between individual neurons [8, 9, 10]. Such causality maps may provide information not just about the excitatory/inhibitory nature of the neurons being observed, but also of hidden neurons that exert influence on recorded neurons [8]. The causality maps which are outputs of such methods are square, asymmetric matrices. We interpret these as weighted, directed networks that we call *hippocampal networks*. Conceivably, applying persistent homology to hippocampal networks will yield additional information about the stimulus space of the recorded neurons, e.g. which neurons played an excitatory role in spatial navigation. However, asymmetric structures are

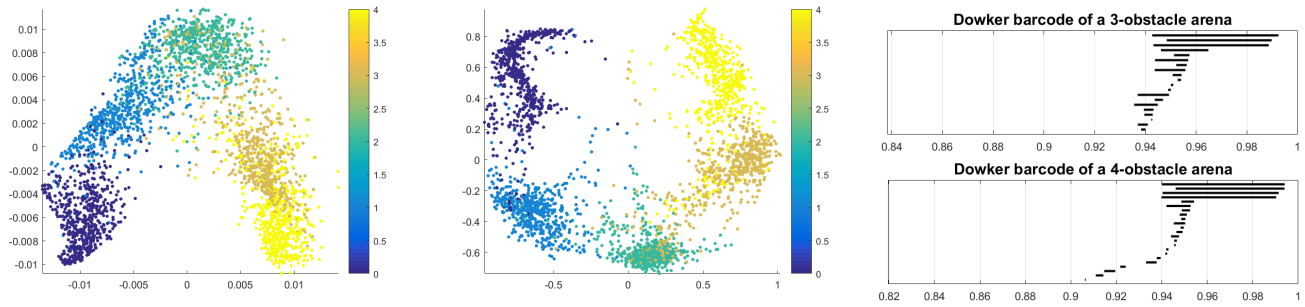

Figure 1: Classical MDS (left) and Isomap (middle) plots of space of persistence barcodes. Integer labels on the color bars indicate the number of obstacles in the arena from which the barcode was generated. Each barcode looks like one of the figures on the right. Notice that the number of long bars corresponds to the number of obstacles. Short bars correspond to noise.

invalid as input to standard PH methods, so the richness of a causality detection method is countered, a priori, by a loss of the insight provided in [4, 5, 6] by homological methods.

Methods. In our work, spike train data for 125 place cells were generated from 3000 simulations of an animal exploring square arenas with 0, 1, 2, 3, and 4 obstacles, respectively, based on parameters used in [6]. The ground-truth topological information for each simulation was the number of obstacles in the corresponding arena.

Directed networks were generated according to the following model: the nodes were the place cells, and the asymmetric weight from cell i to cell j was the fraction of times cell j fired over a threshold h_1 within a time window of $\sim$0.6 seconds *after* cell i had fired over a threshold h_2. The matrices were normalized in order to be interpreted as Markov chains.

To apply persistent homology to these asymmetric networks, we developed a method called Dowker Persistent Homology (DPH) [11] that accepts an asymmetric network as input and computes network invariants called *Dowker persistence barcodes* as output. We applied DPH to the simulated data and compared the resulting barcodes using a metric called *bottleneck distance*. The resulting metric space was then embedded in the plane using classical MDS and Isomap (Figure 1). Videos of a 3D embedding are available on https://research.math.osu.edu/networks/dowker/arenaVid.html.

Results. As shown in the MDS plots of Figure 1, our methods can conclusively extract ground-truth topological information about the stimulus space from directed hippocampal networks, and place networks coming from arenas with the same number of obstacles into well-defined clusters. We propose our DPH method [11] as a general model for uncovering structural information from asymmetric networks.

References

[1] P. Dayan and L. F. Abbott, *Theoretical neuroscience*, vol. 10. Cambridge, MA: MIT Press, 2001.

[2] J. O'Keefe and J. Dostrovsky, "The hippocampus as a spatial map. preliminary evidence from unit activity in the freely-moving rat," *Brain research*, vol. 34, no. 1, pp. 171–175, 1971.

[3] E. N. Brown, L. M. Frank, D. Tang, M. C. Quirk, and M. A. Wilson, "A statistical paradigm for neural spike train decoding applied to position prediction from ensemble firing patterns of rat hippocampal place cells," *The Journal of Neuroscience*, vol. 18, no. 18, pp. 7411–7425, 1998.

[4] C. Curto and V. Itskov, "Cell groups reveal structure of stimulus space," *PLoS Computational Biology*, vol. 4, no. 10, 2008.

[5] Y. Dabaghian, F. Mémoli, G. Singh, L. Frank, and G. Carlsson, "Topological stability of the hippocampal spatial map," in *Front. Syst. Neurosci. Conference Abstract: Computational and systems neuroscience*, 2009.

[6] Y. Dabaghian, F. Mémoli, L. Frank, and G. Carlsson, "A topological paradigm for hippocampal spatial map formation using persistent homology," *PLoS Comput Biol*, vol. 8, no. 8, 2012.

[7] R. Ghrist, "Barcodes: the persistent topology of data," *Bulletin of the American Mathematical Society*, vol. 45, no. 1, pp. 61–75, 2008.

[8] S. Kim, D. Putrino, S. Ghosh, and E. N. Brown, "A granger causality measure for point process models of ensemble neural spiking activity," *PLoS Comput Biol*, vol. 7, no. 3, p. e1001110, 2011.

[9] Z. Li and X. Li, "Estimating temporal causal interaction between spike trains with permutation and transfer entropy," *PloS one*, vol. 8, no. 8, p. e70894, 2013.

[10] S. Emrani and H. Krim, "Effective connectivity-based neural decoding: A causal interaction-driven approach," *arXiv preprint arXiv:1607.07078*, 2016.

[11] S. Chowdhury and F. Mémoli, "Persistent homology of asymmetric networks: An approach based on dowker filtrations," *arXiv preprint arXiv:1608.05432*, 2016.